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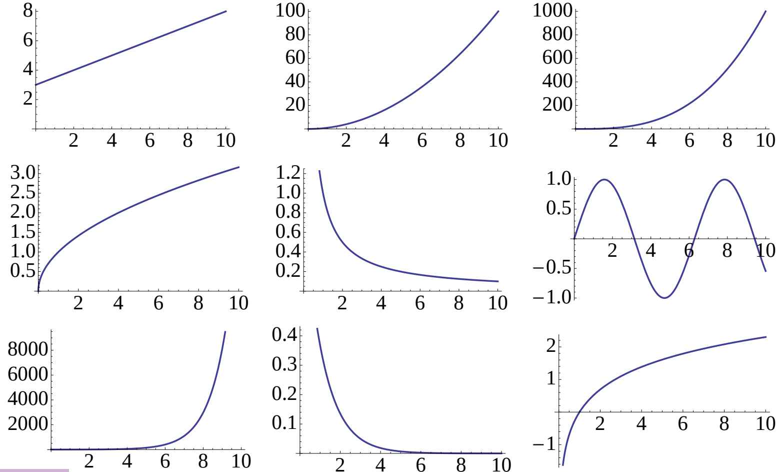
Functions you should recognize on sight



Think Pair Share

What are these functions?

What is the most general equation of each type?

Springs and Elongations

Example: Modeling Spring Elongation

Take your favorite spring. Attach different masses.
How much does it stretch from rest? [Its **elongation**.]

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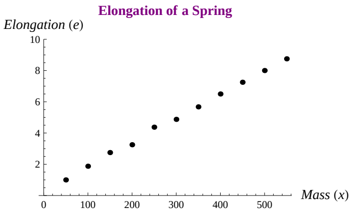
When we plot the data, we get the following **scatterplot**.



mass x	elong y
50	1.000
100	1.875
150	2.750
200	3.250
250	4.375
300	4.875
350	5.675
400	6.500
450	7.250
500	8.000
550	8.750

What do you notice? ______________________

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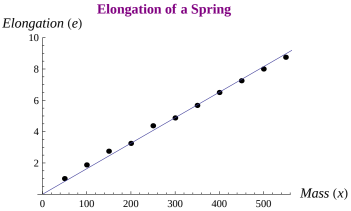

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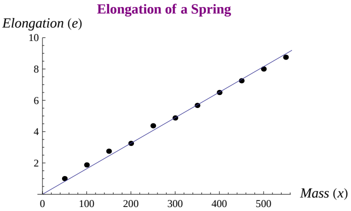
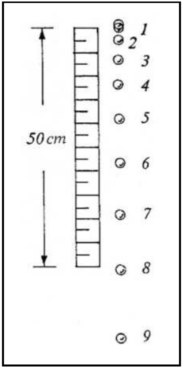


2. Mathematically: **Linear Regression / Least Squares**
   (For another day)

Fitting Gravity Data

Example. Modeling the dropping of a golf ball



Source:
practicalphysics.org

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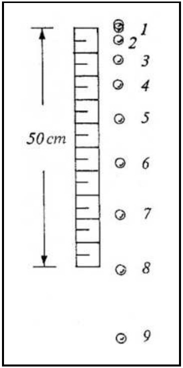

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Let's use an experiment to test the gravity model from last time.

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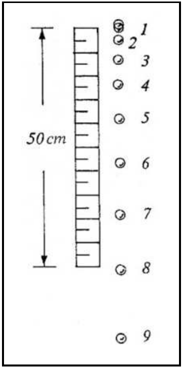

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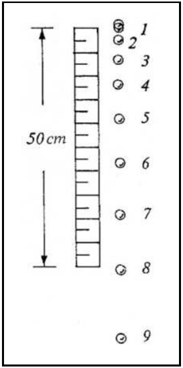

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Let's use an experiment to test the gravity model from last time.

Use a camera to record the position every tenth of a second.

Data would be similar to the table→

t	y
0.0	0.00
0.1	0.25
0.2	0.75
0.3	1.50
0.4	2.50
0.5	4.00
0.6	5.75
0.7	7.75
0.8	10.25
0.9	13.00
1.0	16.00

[Ignore data on p. 25.]
[It's BAD data.]

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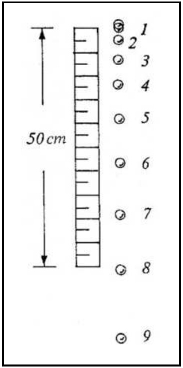

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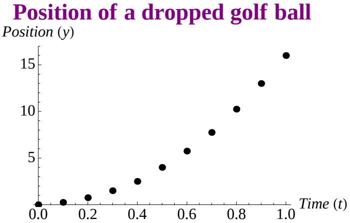
Data would be similar to the table→
It's plotted in the scatterplot below.



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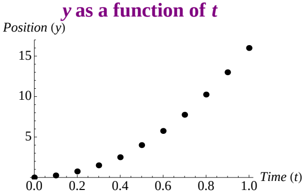
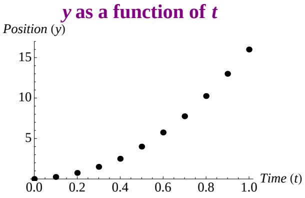
These data seem to fit a $\underset{\text{(type of function)}}{\underline{\hspace{8em}}}$.



Fitting Gravity Data

These data seem to fit a $\underset{\text{(type of function)}}{\underline{\hspace{4cm}}}$. How can we be sure?



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These data seem to fit a $\underline{\hspace{3cm}}$ (type of function). How can we be sure?

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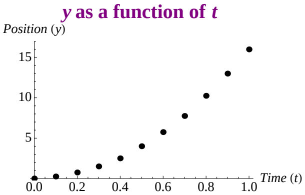

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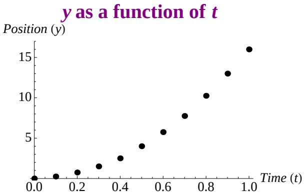

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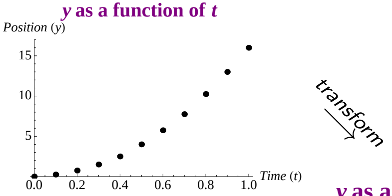

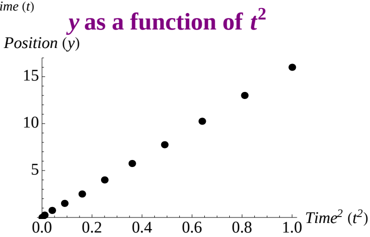

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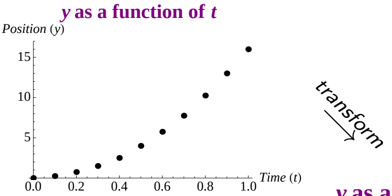

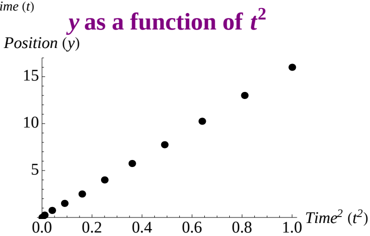

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This implies
$y \approx \_\_ t^2$.

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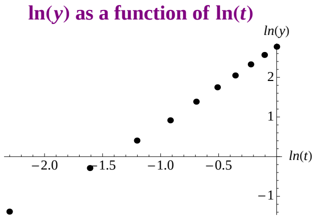

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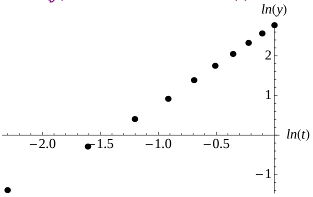
ln(y) as a function of ln(t)



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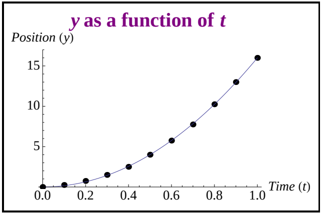
We have determined that our gravity model

$$y(t) = 16t^2$$

appears to model the dropping of a golf ball.



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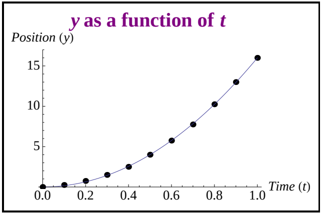

Example. Raindrops

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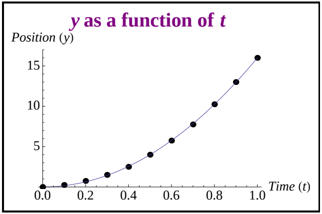

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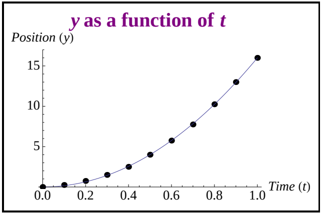

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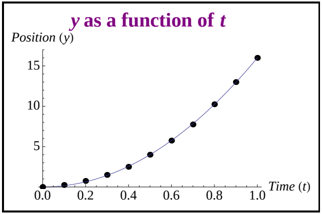

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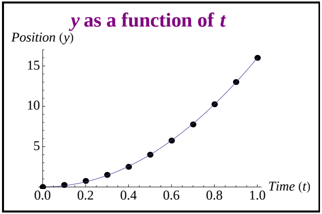

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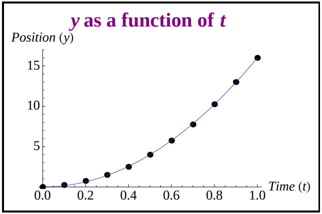

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Even if we have a good model for one situation doesn't mean it will apply everywhere. **We always need to question our assumptions.**

—Extensive gravity discussion in Section 1.3.—

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$P(t) =$ size of population at time t.
$B(t) =$ number of births between times t and $t + 1$.
$D(t) =$ number of deaths between times t and $t + 1$.

Therefore, $P(t + 1) =$ ____________________.

Definitions imply
$P(4) =$
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Assumption: No migration.

Population Growth

Therefore, $$P(t+1) = P(t)\left[\frac{P(t)}{P(t)} + \frac{B(t)}{P(t)} - \frac{D(t)}{P(t)}\right].$$

Under our assumptions, $$P(t+1) = P(t)[1 + b - d].$$

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This implies: $P(1) \quad = \quad$ __________,

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> A model for the size of a population is
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Applying the Malthusian Model

Approximate US Population at: http://www.census.gov/main/www/popclock.html

Example 1. Suppose that the current US population is 317,420,000. Assume that the birth rate is 0.02 and the death rate is 0.01. What will the population be in 10 years?

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Refinement. Approx. US Growth Rate at http://www63.wolframalpha.com/input/?i=US+birth+rate

Resource: Wolfram Alpha, integrable directly into *Mathematica*.

Example 2. How long will it take the population to double?

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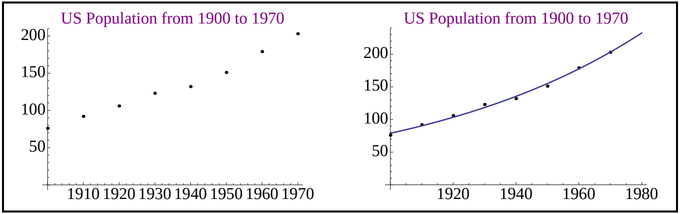
Determining constants of exponential growth

Goal: Given population data, determine model constants.



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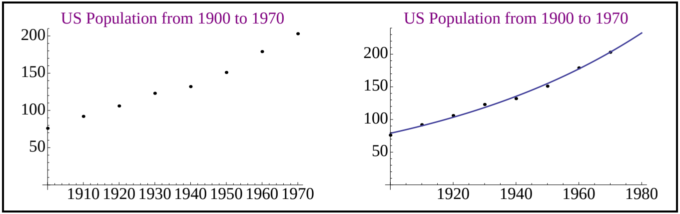

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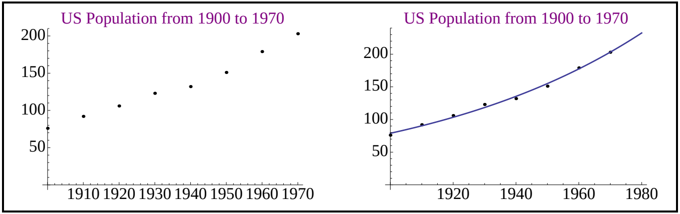

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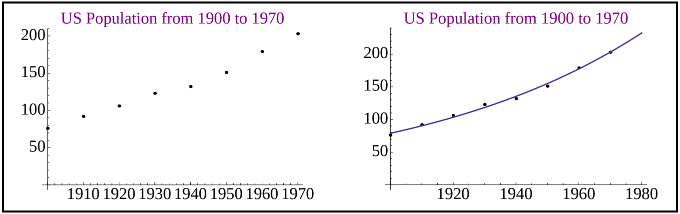

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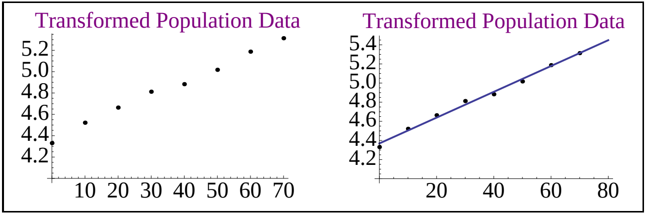

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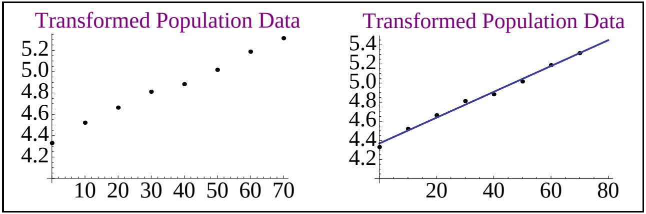

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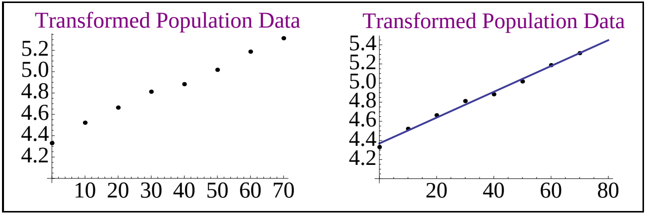

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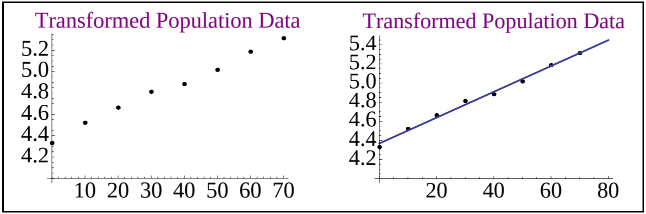

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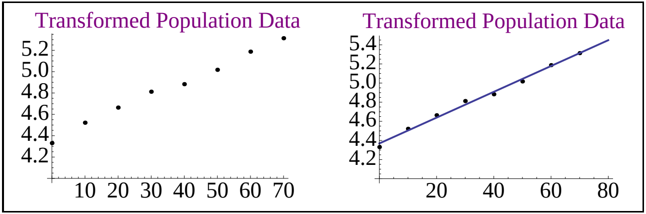

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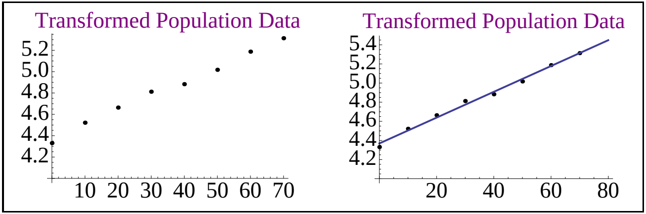

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⋆ Important: Transformations distort distances between points, so verification of a fit should always take place on y versus x axes. ⋆

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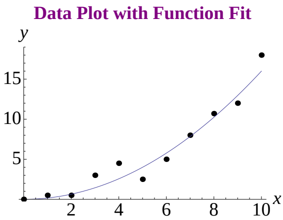

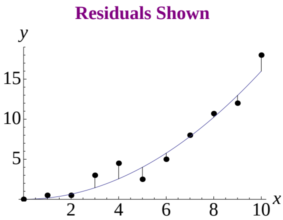

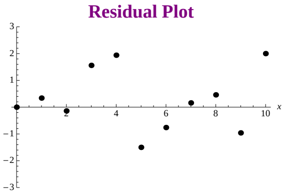

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The structure of the points in the residual plot give clues about whether the function fits the data well. Three common appearances:

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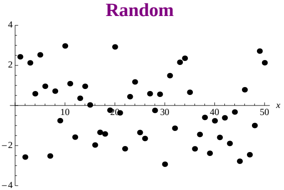

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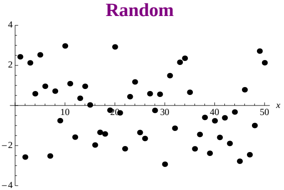

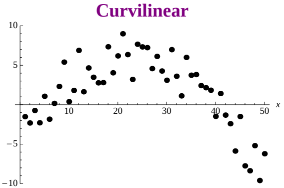

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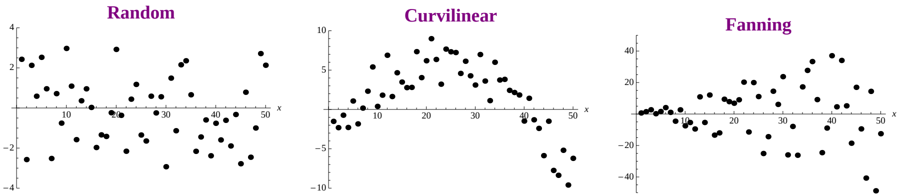

Interpolation vs. Extrapolation

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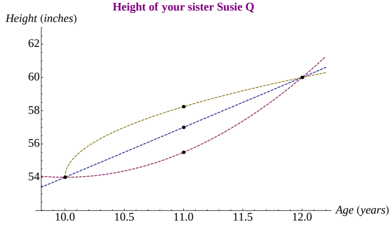
Inserting one or more x-values between known x-values.



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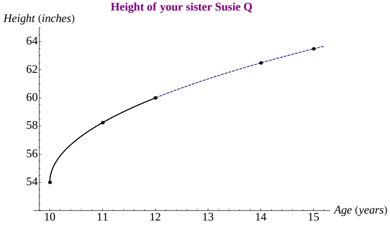
Inserting one or more x-values outside of the range of known x-values.



Interpolation vs. Extrapolation

- The most common method for interpolation is taking a weighted average of the two nearest data points; suppose $x_1 < x < x_2$, then,

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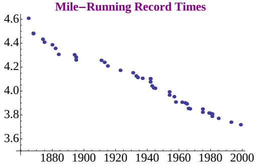
Extrapolation: Running the Mile (p. 162)

Below is a plot of the years in which a record was broken for running a mile and the record-breaking time.



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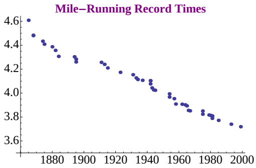

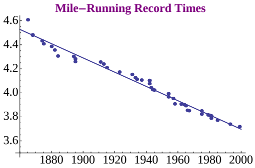

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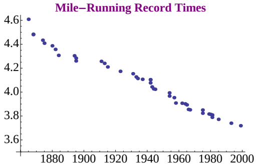

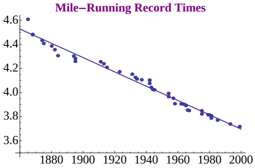

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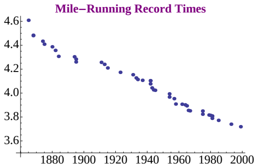

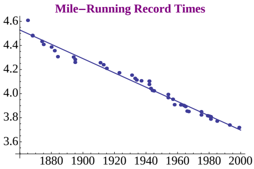

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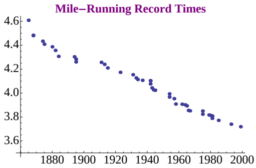

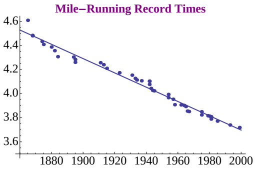

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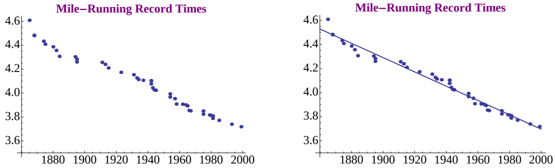

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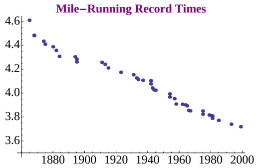

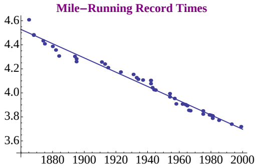

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