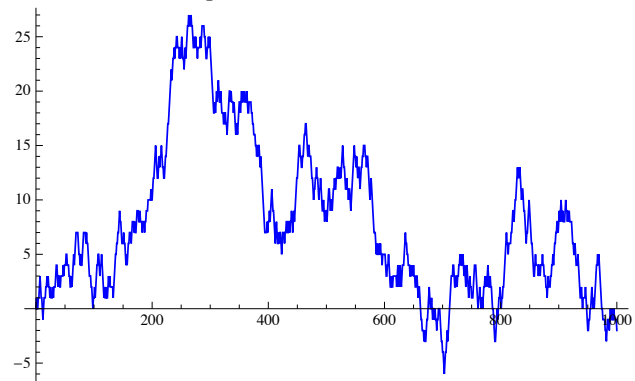


# Random Walk

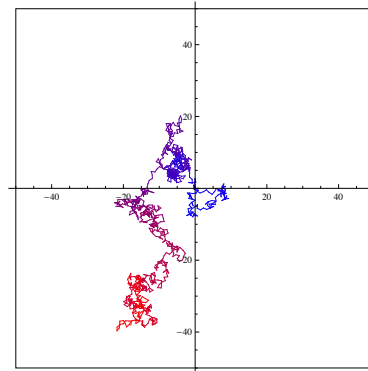
A **random walk** is a sequence of steps, where each step is generated randomly and depends only on its current position.

Random walks can be thought of as a special type of Markov chain.

**Stock prices**



**Movement of a molecule in liquid  
(Wiener process)**



**Genetic drift**

**Diffusion of  
populations**

**Polymers**

**Shuffling of a deck of cards.**

Each state is one of the  $n!$  permutations of the  $n$  cards.

We transition from one state to another by some rule. Perhaps:

- ▶ Moving a random card to a new position.
- ▶ Choosing a pair of random cards and exchanging them.

## Simple random walk

**A drunk in a bar.** A bar patron has had a little too much to drink and it's about time to leave the bar. There is an exit directly to his right and an exit three steps away to his left. The drunk stumbles randomly one step to the left or one step to the right with equal probability.

What is the probability that the drunk leaves via the right door?

What is the transition matrix for this random walk?

What is an equilibrium solution for this random walk?

## Gambler's Ruin

**Win or go home broke!** A gambler starts with \$500 and makes \$1 bets, winning each with probability  $p$ .

The gambler stays until she has made \$100 profit or goes broke.

*Question.* What is the probability that she goes home a winner?

This depends on  $p$ . For roulette:  $p = 18/37 \approx 48.6\%$ :

We can model this with a random walk.

There also exist higher-dimensional random walks.

## Color mixing game

Let's play an interactive Markov chain game.

- ▶ Choose a color. (Red, Orange, Yellow, Green, Blue, Purple)
- ▶ Record the distribution.
- ▶ Do some Markov mixing.
  - ▶ Find a random partner. Announce your colors.
  - ▶ Randomly decide whose color will prevail.  
(Coin flip or Rock Paper Scissors)
  - ▶ Both players now take the winning color.
  - ▶ Repeat many times!
- ▶ Record the distribution at multiple times during the experiment.

What do we expect to occur?

Stand up and make some space to move around.