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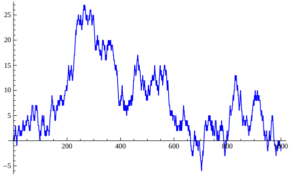
Stock prices



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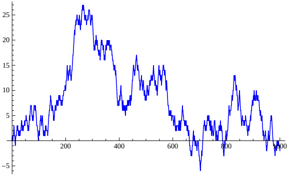
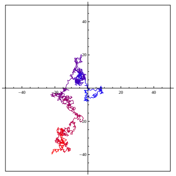
Stock prices **Movement of a molecule in liquid (Wiener process)**





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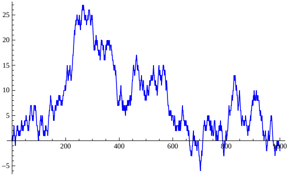

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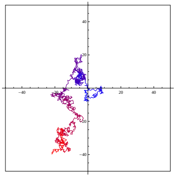

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Diffusion of populations

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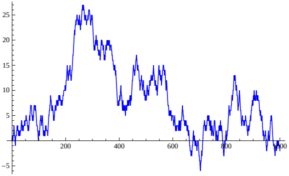

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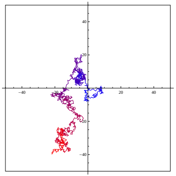

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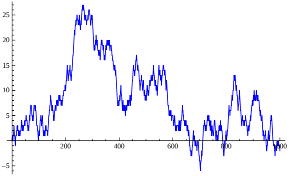

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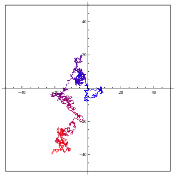

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Shuffling of a deck of cards.
Each state is one of the $n!$ permutations of the n cards.
We transition from one state to another by some rule. Perhaps:

- Moving a random card to a new position.
- Choosing a pair of random cards and exchanging them.

Simple random walk

A drunk in a bar. A bar patron has had a little too much to drink and it's about time to leave the bar. There is an exit directly to his right and an exit three steps away to his left. The drunk stumbles randomly one step to the left or one step to the right with equal probability.

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What is the transition matrix for this random walk?

What is an equilibrium solution for this random walk?

Gambler's Ruin

Win or go home broke! A gambler starts with \$500 and makes \$1 bets, winning each with probability p.
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There also exist higher-dimensional random walks.

Color mixing game

Let's play an interactive Markov chain game.

- Choose a color. (Red, Orange, Yellow, Green, Blue, Purple)
- Record the distribution.

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    (Coin flip or Rock Paper Scissors)
  - Both players now take the winning color.
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What do we expect to occur?

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What do we expect to occur?

Stand up and make some space to move around.