Modeling: Start to Finish

Example. Vehicular Stopping Distance

Background: In driver's training, you learn a rule for how far behind other cars you are supposed to stay.

- Use the two-second rule: stay two seconds behind.

This is an **easy-to-follow** rule; it is a **safe** rule?

Formulation.

State the question. Identify factors. Describe mathematically.

Culminates with a mathematical model.

Mathematical Manipulation.

Determine mathematical conclusions.

Evaluation.

Translate into real-world conclusions. How good is the model?

Formulation

First, we need to state the question (or questions) clearly and precisely.

★ Does the two-second rule mean we'll stop in time?

★★ Determine the total stopping distance of a car as a function of its speed.

Now we need to identify factors that influence our problem statement.

Stopping distance is a function of what?

- v velocity
- t_r reaction time
- a vehicle acceleration / deceleration

Breaking down the problem

Describe mathematically.

Subproblem 1:

Determine reaction distance d_r

Assume speed is constant throughout reaction distance. Total reaction distance is

$$d_r = t_r \cdot v.$$

Subproblem 2:

Determine stopping distance d_b

Assume brakes applied constantly throughout stopping, producing a constant deceleration.

Brake force is $F = ma$, applied over a breaking distance d_b.

This energy absorbs the kinetic energy of the car, $\frac{1}{2}mv$.

Solving $m \cdot a \cdot d_b = \frac{1}{2}mv^2$, we expect

$$d_b = C \cdot v^2.$$

Therefore, the total stopping distance is $d_r + d_b = t_r v + Cv^2$.

Mathematical Manipulation

- We have a mathematical model.
- Did we answer the question?

We need to determine the reaction distance and stopping distance.

- Does data already exist? If not, can we gather data? Get it!

Data is available from US Bureau of Public Roads.

Reaction distance is tabulated in Table 2.4 and shown in Figure 2.14.

The data lie perfectly (**!**) on a line. $d_r \approx 1.1v$.

- Examine methodology of data collection.
- Experimenters said $t_r = 3/4$ sec and calculated d_r!
- Perhaps we should design our own trial?

Function Fitting

- Compiled data is a range.
  - Trials ran until had a large enough sample
  - Then middle 85% of the trials given.
- We're modeling d_b as a function of v^2, so transform the x-axis.
- Do we try to fit to low value, avg value, or high value in range?
  - Goal: prevent accidents!
- Consider the line in Figure 2.15: $d_b \approx 0.054v^2$.
- Up to 60 mph ($v^2 = 3600$), seems like reasonable fit.

We conclude that the total stopping distance is

$$d_{tot} = d_r + d_b \approx 1.1v + 0.054v^2.$$

Check fit: Plots observed stopping distance versus model. (Fig. 3.16)

- Model seems reasonable (through 70 mph).
- Residual plot shows additional behavior unmodeled (Fig. 3.17)

How good is the model?

Is the model accurate?

- Basically yes. Our model gives a slight overestimate of total stopping time through 70 mph.

Is the model precise?

- Yes, the model gives a definite answer.

Is the model descriptively realistic?

- Yes, the model was created based on the physics of stopping.

Is the model robust?

- Suppose that **error in** is -10% for a true speed of $v = 60$. Then $v' = 54$ and the model predicts that stopping distance is $1.1 \cdot 54 + 0.054 \cdot 54^2 \approx 217$ instead of $1.1 \cdot 60 + 0.054 \cdot 60^2 \approx 260$. This is an **error out** of -16%. Our model is not robust.

Limitations and assumptions inherent in our model:

Is the model general? When is it reasonable? What are its limitations?

- Drivers going $\leq$ 70 mph
- Good road conditions
- Applies when driving cars, not trucks.
- Current car manufacturing; revise every five years
- In the future, perhaps there will be no accidents!

Maintain the model

Is the model fruitful? Does it inspire other models?
Can it be widely implemented?

- This line of reasoning can be applied to any situation with constant deceleration.
- Come up with a good rule of thumb for drivers to follow and publicize it. (Next slide!)

Vehicular Stopping Distance

Does the two-second rule mean that we'll stop in time?

- Recognize that a two-second rule is **Easy to implement.**
- The two-second rule is a *linear* rule,
- A *quadratic* rule would make more sense.
- Works up until 40 mph, then quickly invalid! (Figure 2.17)

Come up with a variable rule based on speed.

- It's not reasonable to tell people to stay 2.5 seconds behind at 50 mph and 2.8 seconds behind at 58 mph!
- Determine speed ranges where
  - two seconds is enough ($\leq$ 40 mph)
  - three seconds enough ($\leq$ 60 mph)
  - four seconds enough ($\leq$ 75 mph)
  - Add more if non-ideal road conditions.