Introduction to Oligopoly
Bertrand model as prisoners dilemma: and cartel.

- Oligopoly: study of a small number of firms acting independently of each other but aware of each others actions.
- Firm must consider its rivals actions in making its own profit decisions.

- Non-cooperative Game theory: vo Neuman 1944.

Assumptions:
- consumers are price takers
- firms Homogenious products (price is only information)
- no entry:
- firms collectivly have market power

Consider the following consumer:

$$p = a - bq \quad \rightarrow \quad q_i \frac{(a-p)}{b} \quad \rightarrow \quad p = 10 - \frac{1}{2} q$$

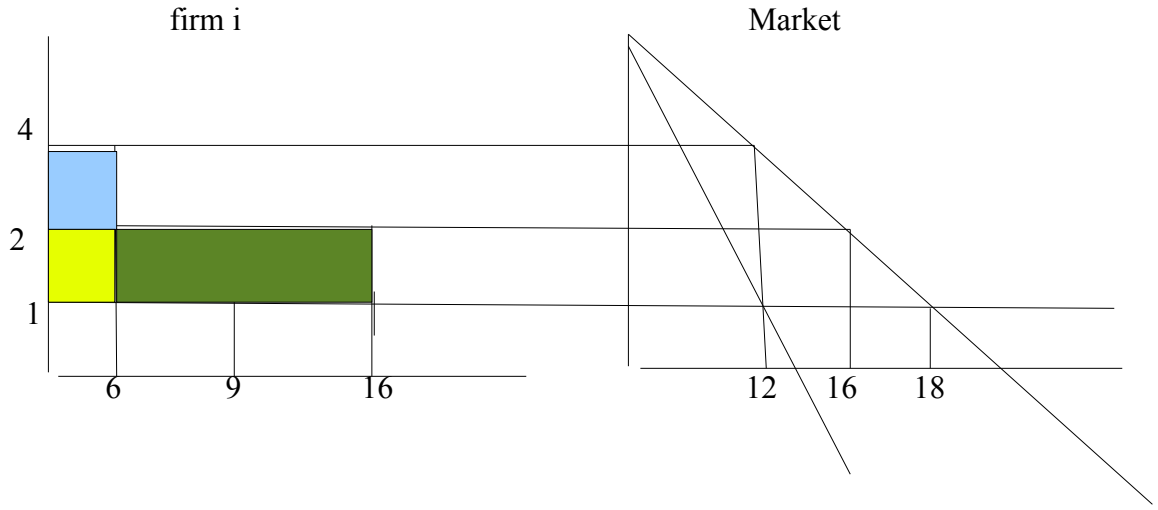
Two Firms [i and j] compete by setting prices:
consumers buy all their output from the firm with the lowest costs
thus the quantity any of our two firms can expect to receive is given by

$$q_i \begin{cases} 0 \text{ if } \quad p_i > p_j \\ \frac{(a-p)}{2b} \text{ if } \quad p_i = p_j \quad \text{i.e. They split the market} \\ \frac{(a-p)}{b} \text{ if } \quad Pi < p_j \quad \text{firm i offers a lower price.} \end{cases}$$

Assume that each firms costs function appears as

$TC = cq$ since there are no fixed costs and constant marginal costs average cost=mc

for simplicity assume MC = c = 1



$\pi_i = (p - AC)q \rightarrow AC = c$

cooperation, act as a monopoly

$p_i = p_j = 4$ $\quad q_1 = \frac{(10-4)}{(2(\frac{1}{2}))} = 6$ $\quad \pi_i = (4-1)6 = 18$

Cheater one firm drops price to 2 stealing all market demand

$p_i < p_j \rightarrow p_i = 3$ $\quad q_i = (\frac{(10-3)}{(\frac{1}{2})}) = 14$ $\quad \pi_i = (3-1)14 = 24$

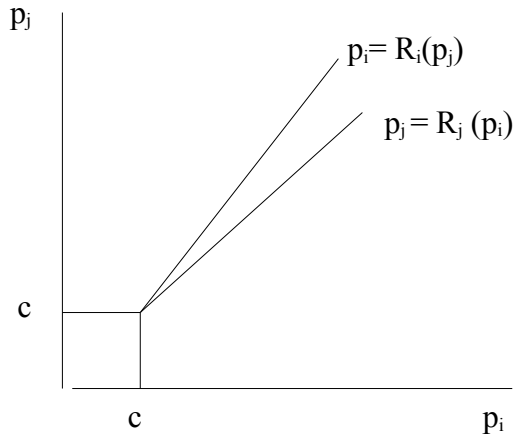
the firm that did not cheat continues to produce cooperative levels of 6 units, yet they sell nothing so they earn a loss.

$\pi_j = (0-1)6 = -6$

Both cheat on each other until prices are at their minimum:

$P_i = p_j = mc = 1$ $\quad q_i = \frac{(10-1)}{(2(.5))} = 9$ $\quad \pi_i = (1-1)9 = 0$

Bertrand Best Response function: (non-cooperative oligopoly Behavior



- **Bertrand**
  Competition in Prices
  Best Response functions are upward sloping.
  Outcome is a Nash Equilibrium at which consumption is efficient, P=MC, even with Barriers to Entry.
  (Since producers are competing away each others profits, they must bid down profit until $\pi = 0$)

As Game (prisoners dilema)

	Cheat	co-operate
Cheat	0 0	-6 24
Cooperate	24 -6	18 18

Four Possible outcomes, each is a representation of a set of actions taken by each player:

(co, co)(co,ch)(ch,co)(ch,ch)

Payoffs:

$\pi_j = \pi_i(ch, ch) = 0$
$\pi_i(ch, co) = 24$
$\pi_i(co, co) = 18$
$\pi_i(co, ch) = -6$

Equilibrium:

Dominate Equilibrium

When one player has the same choice no matter what the rival does
here cheats dominant.

From viewing the payoffs above we can see that regardless of whether the rival cheats or co-operates player i has the same strategy; namely to Cheat.

Nash Equilibrium:

An outcome for which no player would find it benificial to deviate provided that all other players do not deviate from the present strategies played at the Nash outcome.

Finding a Nash: look at available outcomes and see if, we can rule out the any outcome. An outcome can be ruled out if at that outcome a player could increase teheir payofff by choosing some other action.

Start at (co, co) either player could increase their payoff by choosing to cheat.
At some outcome (ch,co) the rival player could increase their payoff by choosing to cheat
At (ch,ch) no player would find it in their best interest to start co-operating.

Thus (ch,ch) is the **Nash Eq**.

Rules for Nash Eq:

an equilibrium in dominant actions is a NE
a NE need not be a Equilirium in Dominant Actions.
A NE need not be unique, there may be multiple NE.
(return latter with Battle of the Sexes)

Maintaining Cooperation: Creating and enforcing a Cartel, Resolving the Prisoners Dillema

- When firms are able to cooperate we have cooperative Oligopoly behavior: and a theory of Cartels.

Cartels: may involve; price fixing, output controls, bid rigging, allocation of customers, alloation of sales by product or teritory, establishment of trade practices, or common sales agencies. (Weis 1987)

- When co-operation Breaks down we have Non-cooperative Oligopoly behavior; Bertrand (or Cournot)
- Cooperation could be achieved if parties were able to create and enforce binding contracts between themselves
- Anti-Trust Legislation makes Contracts Illegal:

Sherman Anti-Trust Act (1890)

"Every contract, combinationin the form of a trust or otherwise, or conspiracy in restraint of trade or commerce among several states, or with foreign nations, is declared to be illegal" (Shy p.90)

- Price fixing can result in fines equal to Treble-damages. Some debates ask whether treble damages lower the market price. Theorists have shown that under threat of treble damanges the market price

is higher than what it would otherwise be charged by the cartel without anti-trust enforcement. (shy 90)

- Anti-trust law may be difficult to enforce (suing the Rule of Reason) since, for price fixing contracts, it is not clear what an reasonable price is, difficult to view MC functions to determine what was a high markup. (as a consequence courts began using the Per Se rule- which states that any contract fixing price was illegal regardless of intent- the logic here is that a contract would not be formed unless it had an affect on prices)
- The Sherman Anti-trust act was successful at eliminating major cartels from American Markets. Nowadays most noticable cartels OPEC and IATA are international and can't be challenged for very visible price fixing agreements.

As we have seen, In a one-shot game Cooperation can not occur.

Yet, in a repeated game co-op may occur.

Factors that may facilitate the formation and enforcing of a cartel:

- Ease of communication and negotiation between firms, existing Trade Associations.
- Few firms involved
- highly concentrated industry
- nearly identical product, easy to price
- Prices are widely known.

Cooperation can be maintained if Cheaters can be **Detected and Punished** in this way agents create their own contract and enforcement.

1) Detection of cheating:
   - in a game of n>2 players detecting cheating may be easy but detecting WHO cheated may be hard.
   - Prices may fluctuate independently (difficult to tell if cheating has occurred or market conditions have changes)
   two reasons for price to fluctuate: supply controls-state of demand.

Example: OPEC: the price of oil has been rising to spectacular levels in recent months. OPEC contends that it is not they who have restricted supply, thereby driving up prices but that the rising price of OIL is being driven up by Speculators on Wall street.

- Alternatively does a fall in the price of oil mean that there is a cheater in OPEC or that demand has slacked off?

These problems; where chages in price may be ambiguous or detecting who cheated is difficult may lead to a blanket punishment to all members of the cartel. Or a

Trigger Price: a threshold price below which all agents will stop cooperating. This trigger price signals an emediate dissolution of the cartel regardless of wheterh the price fall was due to cheating or market conditions. In one way this trigger price strategy helps to maintain trust in the cartel. In another it elliminates the cheaters gains quickly (reducing their incentive to cheat in the first place) and provides for repentance: if once the cheater stops cheating (or market conditions turn around) the price rises and the cartel reforms.

2) Price changes may be easy to observe but other methods of competition may not.
   - Secret discounts
   - Pricing of Trade-ins
   -Quality or Service.

By enhancing the product allong dimensions of quality the firm effectivly lowers price. This is because the consumer is paying the same price for a higher quality good.

When competition takes place on increasingly subttle dimensions other than price we refer to this as **The law of increasing opaqueness.**

Example:airlines during regulation by civil Aeronautics Board effectivly had a government created cartel. Soon airlines began competing along other dimenstions
prettyest stewardesses, (then gender law) , leg room, meals, schedules etc..

Punishment

- Police threaten that they will let them go! Prisoner knows that if the cops let them go, their friends will assume that they ratted, and punish them.

- (Present versus future profits: gain now from cheating may be offset by losses in the long term that are the outcome of competition. The tradeoff between present and future profits will in turn factor into the cheaters decision. Think as society becomes more chaotic you worry less about the future because the present is so critical to survival. One loaf of bread is worth more today than it will be 10 years from now if you are starving. Also, this in my mind generates a theory of inflation in a market structure dominated by cartel like firms: as interest rates fall (the discount rate)
- Example: consider a firm with choices to cheat or cooperate. The game last two periods. If the firm cooperates in period 1 and period 2 the profit earned is
  - 55 + 55 = 110
- If the firm cheats the firm gains a profit this year of 75 and then cooperation breaks down, so that in the second year both firms cheat and the firms profit is 25
  - 75+25= 110

- It would appear that the firm is indifferent between cheating and cooperating as future losses will offset current gains. And on net the firm will be no better off for cheating.
- Now consider the role of the discount rate (interest rate) that allows us to evaluate the present value of future profits (or the future value of present profits.
- If the firm cheats in year 1 its net gain comparing cheat and cooperate strategies is
  - 75-55 = 20

  if the firm maintains cooperation its net gain in year two comparing strategies is
  - 55- 25 = 30

- having 20 today will be equivalent to having 30 in year two only if the disocunt rate is .5
  - 20(1+.5)=30

  in this case the firm is still indifferent between cheating or cooperating.
- If the discount rate were to fall, then present profits would be worth less in the future
  - 20(1+.25)=25 and the firm would prefer to cooperate rather than cheat

  if the discount rate were to rise, then present profits would be worth more in the furture (consider them invested) and as a consequence the firm would prefer to cheat Now.
  - 20(1+.60)= 32

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- alternatively you could do it via discounting 30 back one year at rates above, below and equalt to .5.
- disocunting 30 back 1 year at a lower rate 1.25 we have 30/1.25=24. lossing 30 in the future is the same as gaining 2 now. The firm only gains 20 not and so will not cheat
- discounting 30/1.60 =18.75. lossing 30 in the future is equivalent to gaining 18.75 now. The firm gains 20 from cheating now, and so will cheat.]]]]]]]]]]]]]]]]]]]]]]]]]]]]]]
- (as interest rates rise current profits become more attractive than any given future sum thus cartels will break down when interest rates rise, during recession more competition in terms of prices will be evident, in good times when interest rates are low, cooperation will be more easily maintained and higher prices will be exhibited. Lowering interest rates facilitates cartel cooperation and effects in terms of rising prices)

Meeting competition Clause: long term supply contract or adversiting guarantee that if any buyer can find the good elsewhere for a cheaper price the firm will match and or beat the rivals offer.

MCC has the effect of using the consumer to indicate who is cheating. And by emediatly undercuting the cheating rival, eliminates any short run gains the rival might have expected from cheating. This punishment is then guaranteed and no rival cheats. What looks on its face like competitive behavior is actually a means by which the cartel is held together.

"im lowering prices prices out the door, prices lower than any competitor we undercut anyone and everyone and give you the kitchen sink, am I crazy? How does he do it?" Answer: he is part of a cartel!!

Mob uses capital punishment.

Caveates:

Though cooperation may be observed when games are repeated infinitely In FINITE games where there is some natural end cheating will still dominate.

The result of the last game is the same as the one shot game- no punishment can ensue so dominant strategies exist-and cheat cheat is the outcome.

The second to the last game is played with the last game outcome known. Everyone will cheat in the last game, so in the second to the last game firms will try to get a short run gain by cheating. Both firms think alike so cheat, cheat is the dominant outcome.

The logic continues all the way to the first game in which cheat cheat must be the logical choice.

Example: Three panel Prisoners dilema:

TIT FOR TAT

Eye for an Eye. Performed the best in computer simulations of (infinitly?) repeated Prisoners dilemas. (robert Axelrod of U of Michigan)

Tit for tat performed better than any other strategy, even though it did nto and could not beat any of its rivals head on. At best, tit 4 tat ties its rivals. But tit for tat always comes close. So avoids huge losses implied by other strategies.

Tit for tat usually inspires cooperation and avoids exploitation.

However in the real world when actions can be misinterpereted as aggression, tit for tat can lead to an endless cycle of agression. Any mistake echoes back and forth.

Yet,Successfull cooperation occurs in the real world even if the game has a finite number. This may be the case if "nice" is a good strategy to reap benefits of cooperation until your true nature is known. Or if the exact number of games (though finite) is unknown)

Other common Game Forms:

Battle of the Sexes (coordination game)

-dominant actions do not exist for many applications

		Racheal Opera	Racheal Tennis
Dan	Opera	2, 1	0, 0
	Tennis	0, 0	1, 2

Intuition: Agnet gain utility from being together and earn no utility when apart However, each has preferences about HOW they are together.

Here Dan Prefers Opera and Racheal Prefers Tennis

Example: product that operate on different standards (Mac, windows. VHS Beta, HDTV, Blue X) two interacting firms (or consumers).

A firm has an accounting department and an adversiting department; the accountants would like to use Windows Excel spreadsheet the design department would like to use MAC design software. The firm must buy new computers for the company. If each gets their individual way, the offices can not communicate. Which set of computers should the firm buy?

No Equilibrium in dominant actions:

To prove this show that at least one player does not have a dominant action:

$\pi^1(O,O)=2>\pi^1(T,O)$ But $\pi^1(T,T)>\pi^1(T,O)$

if Racheal chooses Opera Dan would like to choose Opera.

if Racheal chooses Tensis Dan would like to choose Tenis.

There are Two **NE:**

Equilibriums at which no player would choose another action, assuming that their rivals action was given.

$$\pi^d(O,O) > \pi^d(T,O)$$
$$\pi^r(O,O) > \pi^r(O,T)$$

Assuming that the rival's choice did not change neither Racheal nor Dan would find it rational to deviate from Opera, Opera. Thus (O,O) is a NE.

$$\pi^d(T,T) > \pi^d(O,T)$$
$$\pi^r(T,T) > \pi^r(T,O)$$

Neither Dan nor Racheal would deviate from (T,T) also a NE

we see:

- No dominant action
- Multiple Nash Equilibrium.: the equilibirum is NOT unique, reducing the predictive power of the model.
- (all DA are NE but not all NE are DA)

Rearranging the payoffs we can create a scenario in which there is Non-existence of a Nash Equilibrium.

Reflecting say 30 years of marriage, Dan is MORE inclined to spend time with Racheal. But Racheal is less inclined to spend time with Dan.

		Racheal	
		Opera	Tennis
Dan	Opera	2 0	0 2
	Tennis	0 1	1 0

We have lost our NE.

At (O,O) Racheal will defect to Tennis

At (T,T) Racheal will defect to Opera

At (O,T) dan will defect to tennis

At (T,O) Dan will defect to Opera.

Best Response Function: a function that searches and determines a players best response to any given set of possible actions choosen by the opponent. Best Reseponse functions allow us to search for NE.

After 30 years **Original**

$$R^d(a^r) = \begin{cases} \textbf{O if } a^r = O \\ \textbf{T if } a^r = T \end{cases} \qquad R^d(a^r) = \begin{cases} \textbf{O if } a^r = O \\ \textbf{T if } r^r = T \end{cases}$$

$$R^r(a^d) = \begin{cases} \textbf{T if } a^d = 0 \\ \textbf{O if } a^d = T \end{cases} \qquad R^r(a^d) \begin{cases} \textbf{O if } a^d = 0 \\ \textbf{T if } a^d = T \end{cases}$$

as you can see the reactions functions equal each other at either (O,O) or (T,T) in the original game. But in the after 30 years game the reaction functions never equal each other. If dan picks O then racheal picks T and if racheal pics tenis then dan picks tennis but then racheal would pick opera..

Welfare:

much like in the market structures we are used to anylizing we now have concepts of an equilibrium.

We allso need a criterion by which we can compare different equilibria.

Pareto Efficiency Criterion:

An outcome is said to be Pareto efficient if at that outcome it is impossible to make any agent better off without making another agent worse off.

i.e. Is is impossible to raise the payoff for one party without decreasing the payoff of the rival.

More refined definitions: (shy p22)

Pareto dominate: an outcome *a* Pareto dominates another outcome *b* (is pareto superior) if

a) every player is better of or at least as well of at *a* rather than *b*

and

b) there exists at least one player which is strictly better off at *a* compared to *b.*

the outcome (peace, Peace) is Pareto Superior to (War, War) . i.e. Cooperation pareto dominates cheating.

An outcome is said to be Pareto Efficient if there is no other outcome which Pareto dominates it.

Thus the dominant actions equilibrium, (War, War) or (cheat, cheat) is not Pareto Efficient.

Incidently: an outcome is called **Pareto noncomparable** if for one player *a* is better than *b.* but

for the other player *b* is better than *a.*

i.e. (cheat, cooperate) and (cooperate, cheat) are Pareto noncomparable.

Continuation Extensive form and sub-Perfect Nash

Extensive Form Games:

We have been discussing games in which players move at the same time, once and for all.

Extensive form games, Players can move at different times and more than once.

Extensive form games are descriped in the following way:

- node: point of decision making, first decision is the starting node.

Terminal node: a possible outcome of a series of decisions.

- Decision making tree: a chart showing the possible nodes, and terminal nodes in sequential order.
- Branches link a decision node to its successor nodes.
- (list of N$\geq$ 1 players, i = 1,2,3…N)
- for each node a definition of the player entiled to decision making
- for each player a specification of I's action set at each node that player I is entitled to choose an action.
- Specification of the payoff to each player at each terminal node.

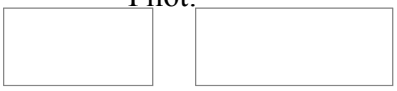
Example:

Pilot Terrorist

A flight from Mineapolis,

Terrorist board flight and tells pilot to fly to Utah or he will blow up the plane.

- Pilot:



Ter. moves: fly to Utah Fly to NY

	Bomb	No Bomb	Bomb	No Bomb
Pilot	$\pi = -1$	$\pi = 1$	$\pi = -1$	$\pi = 2$
Terist	$\pi = -1$	$\pi = 1$	$\pi = -1$	$\pi = 0$

Action set at node 1:

Pilot: (A_I= Utah, NY)

Action set at nodes IIa and IIb:

Terrorist: (A_{Iib}: Bomb, NB)

Strategy: s^i the outline of the possible actions taken by each agent.

A complete list of actions, one action for each decision node that the player is entitled to choose an action.

Terrorist Strategy:

(B,B), (B, NB), (NB, B), (NB, NB)

(first corresponds to the terroists actions in IIa, an the second in IIb)

Pilot Strategy:

(NY, (B,B)), (NY, B, NB), (NY,(NB,B), (NY, (NB, NB)),

(UT, (B,B)), (UT, B, NB), (UT,(NB,B), (UT, (NB, NB)),

Transform in to a normal form game and search for NE:

(possibly skip)

Terrorist

Pilot	(B,B),		(B, NB),		(NB, B),		(NB, NB)	
UT	-1	-1	-1	-1	1	1	1	1
NY	-1	-1	2	0	-1	-1	2	0

The Nash Equilibriums are multiple.

(NY, (B, NB))

(NY, (NB, NB))

(Utah, (NB, B))

(note: UT, (NB, NB)) Is not Nash because pilot would defect to NY(NB,NB)

so it is best in row when looking from the terrorits point of view nad best in columb when viewing the pilots)

(note: the game is not set up to account for the phsychological utility of the terrorist when he gets his 12 virgins in Utah heaven, thus there is no glory in suicide, the ideology of glory in martyrdom creates this utility and makes the irrational possible, thank god for faith eh)

Refining Nash Equilibriums:

Subgames and Subgame perfect equilibrium:

Subgame perfect equilibriums will satisfy all our requirements for NE plus some additional constraints.

This will improve prediction by eliminating, some possible (but undesirable) NEs.

Example: if you were to view the game you would likely consult the Pilot to fly to NY since the Terrorists payoff is greater when under the NY node the terrorist chooses to NOT BOMB (Payoff of 0 for NB and –1 for bomb.

And the Pilot (assuming the terrorits NB) gains a utility of 2 from NY and 1 from UT.

The terrorist payoffs at the terminal nodes indicate that he will choose NB in either case.

The terrorist appears to be playing what is called an *incredible threat*. Since from the terrorists perspective it is unreasonable to Bomb, the threat in not credible under the current structure of utility payoffs.

(comment on how utility was formulated and faith in god)

to see this:

Subgame: a decision node from the orignal game along with the decision nodes and terminal nodes directly following this node. A subgame is subgame proper if it differs from the original game

The terroroist pilot game has theee subgames and two subgame propers

Two subgame propers are as follows:

Ter. moves: fly to Utah Fly to NY

	Bomb	No Bomb	Bomb	No Bomb
Pilot	$\pi = -1$	$\pi = 1$	$\pi = -1$	$\pi = 2$
Terist	$\pi = -1$	$\pi = 1$	$\pi = -1$	$\pi = 0$

Our refined definition of equilibrium:

Subgame perfect equalibrium: outcome is SPE if it induces a NE in every subgame of the original game.

The outcome (NY(NB,NB)) is our subgame perfect equilibrium.

(we have used backward induction to find this)

(once we have a set of NE we just find the NE that works for all subgames.)