Notes on the Cournot Model.
Cournot:

Augustin Cournot French mathematician, 1838.

We will assume that firms derive a strategic function to maximize profit through the choosing of output. Output is considered homogeneous and firm have identical costs structures. There are barriers to entry in the initial model so that a market of only two firms will be analyzed. Later we will relax this assumption showing how the equilibrium outcome changes for any number of firms.

Demand: $P = a - bQ$

1. Firm total Costs: mq_i

Each firm is interested in maximizing its profit:

$$\pi = (a - bQ)q_1 - mq_i$$

In the two firm case: $Q = (q_1 + q_2)$
The profit function for the representative firm, (here firm 1) may be written;

$$\pi_1 = (a - b(q_1 + q_2))q_1 - mq_1$$

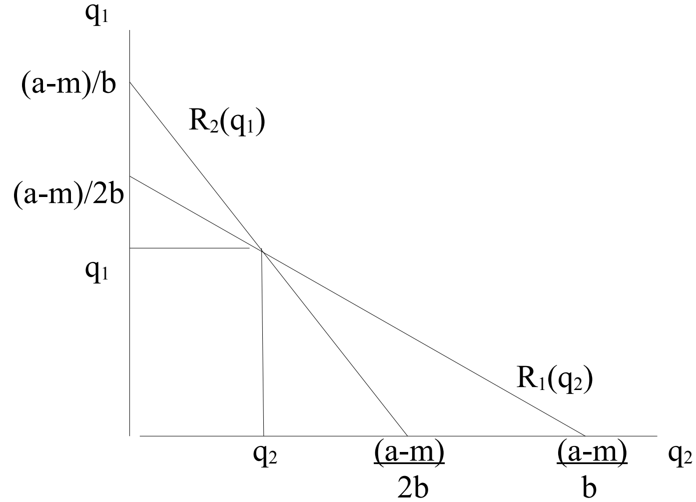
$$\pi_1 = aq_1 - bq_1^2 - bq_2 q_1 - mq_1$$

Differentiation yields the familiar result that each firm sets its output such that MR=MC

$$\pi_1' = a - 2\mathrm{b}\mathrm{q}_1 - bq_2 - m$$

Setting this equation equal to zero and solving for q_1 yields a **Reaction Function** for firm 1**:**

$$q_1 = R_1(q_2) = \frac{(a-m)}{2\mathrm{b}} - \frac{q_2}{2} \quad \text{since firms are identical} \quad q_2 = R_2(q_1) = \frac{(a-m)}{2\mathrm{b}} - \frac{q_1}{2}$$



Nash Equilibrium is found where the Reaction functions intersect, where $q_1(R_2) = q_2(R_1)$. The Nash equilibrium level of output for firm 2 is then;

$$q_2=(a-m)/3b$$

Since $q_1 = q_2$ we have

$$q_2=q_1=(a-m)/3b$$

The Cournot Nash equilibrium level of output for the market as a whole is then;

$$Q=nq=(\frac{n}{(n+1)})(\frac{(a-m)}{b})$$

Now that we have determined the Cournot Nash Equilibrium quantity of output we can determine the price. Simply buy referring to the Demand curve:

P= a – b (Q)

$$P=a-b[2\frac{(a-m)}{(3b)}]$$

$$P=a-[\frac{n}{(n+1)}(a-m)]$$

Cournot equilibrium price

$$P=a\frac{(n+1)}{(N+1)}-\frac{an}{(n+1)}-\frac{nm}{(n+1)}$$

$$p=\frac{(a+nm)}{(n+1)}$$

- Since $P\neq MC$ **We have the presence of DWL,**

Varying the number of firms (n):

$$Q=(\frac{n}{(n+1)}\frac{(a-m)}{b})$$

When this infinite firm case is plugged into our price equation it can be shown that as the number of firms (n) grows large the price tends toward that of the PC case, there P=MC

$$P = a - b\left[\frac{n}{(n+1)}\frac{(a-m)}{(b)}\right]$$

$$p = \frac{(a+nm)}{(n+1)}$$

$$P = \frac{a}{(n+1)} + \frac{n}{(n+1)}m$$

As n to infinity;

$$P = m$$

Notes on Product Differentiation in the Cournot Model:

Our Cournot model can be adapted to product differentiation, will specify a demand curve for firm i:

$$p_1 = a - b_2 q_2 - b_1 q_1$$

We will let the parameters b_2 and b_1 be defines in the following way:

$$b_2 = \frac{dp_1}{dq_2} \quad \text{and} \quad b_1 = \frac{dp_1}{dq_1}$$

Assuming a total cost function with constant marginal costs;

$$c(q_1) = F + cq_i$$

Firm one will then maximize the following profit function.

$$\pi_1 = (a - b_2 q_2 - b_1 q_1) q_1 - cq_i$$

Differentiation yields.

$$\pi_1{}' = a - b_2 q_2 - 2\mathrm{b}_1 q_1 - c$$

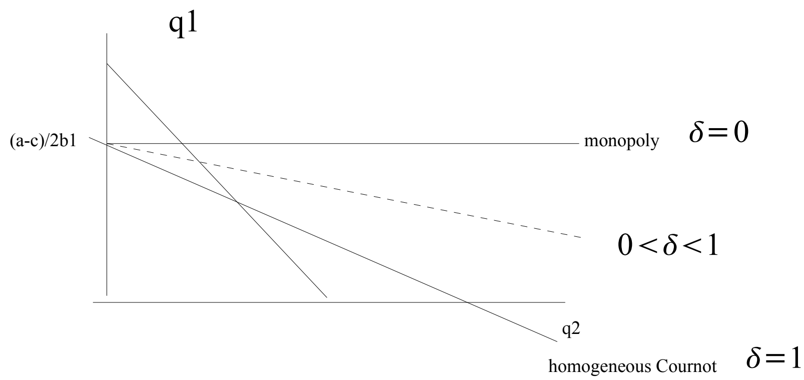
Solving for firm 1's profit maximizing level of output:

$$q_1 = \frac{(a-c)}{2} b_1 - \frac{b_1}{b_2}\frac{q_1}{2}$$

Mapping this reaction curve against firm 2's reaction function reveals the influence of the degree of product differentiation on Nash equilibrium in the Cournot model.

The ratio $\frac{b_2}{b_1}$ is our approximate measure of differentiation δ and we know that $0 \le \delta \le 1$

Where $\delta = 0$ indicates perfect differentiation and $\delta = 1$ indicates that goods are homogeneous.


q1
(a-c)/2b1
monopoly
$\delta=0$
$0<\delta<1$
q2
homogeneous Cournot
$\delta=1$